Int J Theor Phys (2008) 47: 2904–2910
DOI 10.1007/s10773-008-9723-0

A Cyclic Model of Universe with Energy Exchange between Radiation, Matter and Vacuum Energy

Guang-Wen Ma · Jing-Yuan Ma

Received: 24 January 2008 / Accepted: 20 March 2008 / Published online: 1 April 2008

Abstract We further extend the cosmological scenario with energy exchange by Barrow and Clifton and our previous work to the more complex case with energy exchange between three fluids: radiation, matter and vacuum energy. By prescribing the form of energy exchange function, we construct an infinitely cyclic cosmological model, in which the universe undergoes an endless sequence of cosmic epoch and each consisting of expansion and contraction, and the cosmological parameters, such as the Hubble parameter H, deceleration parameter q, transition red-shift Z_{T}, and densities ρ_r, ρ_m, and ρ_Λ are consistent with the present observed values.

Keywords Cosmology · Energy exchange · Cyclic model

1 Introduction

Continually growing evidence indicates that the universe is undergoing a phase of accelerated expansion at the present epoch and the universe is spatially flat [5, 14, 15, 18, 23–25, 28]. This implies that there is sufficient component of dark energy with exotic negative pressure in the universe and the present total energy density of the universe is just equal to the critical density: $\rho_0 = \rho_c \equiv 3H_0^2/8\pi G$. Recalling that the cosmological constant was employed as a kind of "cosmic repulsion" by Einstein to construct a quasi-static cosmological model, as the most natural and simplest candidate of the dark energy, theorists again think of the cosmological constant. However, this would suffer from both old and new cosmological constant problems [10, 36, 37]. In order to overcome these difficulties, various versions of variable cosmological "constant", or decaying vacuum energy, have been proposed in succession. These versions can be roughly grouped into three types. In the first

G.-W. Ma (✉)
Department of Physics, Zhengzhou University, Zhengzhou, Henan 450001, China
e-mail: gwma@zzu.edu.cn

J.-Y. Ma
Shanghai Institute of Applied Physics, Chinese Academy of Science, Shanghai 201800, China

type, the authors specified respective vacuum decay law based on different argument, such as $\Lambda \propto R$ [3], $\Lambda \propto H^2$ [13, 19], $\Lambda \propto \rho_m$ [33], $\Lambda \propto \ddot{a}/a$ [1, 2], and so on. Here $\Lambda = \Lambda(t)$ is the dynamical cosmological "constant", R the Ricci tensor, H the Hubble parameter, ρ_m the matter energy density and $\ddot{a} = \mathrm{d}^2a/\mathrm{d}t^2$ the acceleration of the universe. In another type the authors assumed that vacuum energy has been constantly decaying into cold dark matter, hence the evolution of the energy density of the cold dark matter deviated from the standard case by a positive small constant $\varepsilon : \rho_m = \rho_{m0}a^{-3+\varepsilon}$ [4, 34]. The third type, which was proposed by Barrow and Clifton, was a cosmological scenario with energy exchange [7, 11]. In a series of papers, Barrow, Clifton and other cooperators considered various forms of the energy exchange function and found corresponding cosmological solutions [6, 7, 11, 12]. In our recent work [21], we extended the Barrow-Clifton's scenario from the case of two fluids to the case that universe containing radiation, matter and vacuum energy with energy exchange between radiation and vacuum, as well as between matter and vacuum.

On the other hand, owing to no satisfactory explanation to be offered for the "beginning of time" and the initial conditions of the universe, an endless cyclic model of universe is desired by people all along. There are also a variety of scenarios [8, 9, 16, 20, 29–32]. The old cyclic model has been observationally ruled out. A new cyclic scenario proposed by Steinhardt and Turok [20, 29, 30, 32] and Boyle, Steinhardt and Turok [9] recently was based on a scalar field φ with a particular potential $V(\varphi)$. In fact, such a scalar field was just some type of quintessence or phantom as a source of the dark energy. In their cosmological scenario with energy exchange, Barrow and Clifton also gave a class of the cyclic models [7, 11], however these models were quasi-cyclic and could not avoid the "beginning of time" in principle actually.

Now that the scenario with energy exchange not only can explain the accelerating expansion of the universe without invoking a scalar field, but also is hopeful to solve the cosmological constant problem, it is worth trying to construct a cyclic model of the universe in this scenario. This is just the motivation of the present work.

2 A Brief Retrospection of the Energy Exchange Scenario

For the need of extension later, let us first briefly retrospect Barrow and Clifton's work. For the sake of convenience later, here we slightly change some symbols in the original text without changing their meaning. Barrow and Clifton consider a spatially homogenous and isotropic FRW universe containing two fluids with equations of state

$$p_1 = (\gamma_1 - 1)\rho_1, \qquad p_2 = (\gamma_2 - 1)\rho_2 \tag{1}$$

and introduce the energy exchange function s by

$$\dot{\rho}_1 + 3H\gamma_1\rho_1 = s, \qquad \dot{\rho}_2 + 3H\gamma_2\rho_2 = -s \tag{2}$$

In general, s could be any function of H, ρ_1, ρ_2, t and the cosmic scale factor $a(t)$. The expression (2) is also rewritten as the integral form

$$\rho_1 = \frac{1}{a^{3\gamma_1}}\int sa^{3\gamma_1}\mathrm{d}t, \qquad \rho_2 = \frac{1}{a^{3\gamma_2}}\int sa^{3\gamma_2}\mathrm{d}t \tag{3}$$

After the form of the function s is prescribed, the dynamical evolution of the universe can be found from (2) and the cosmic dynamical equation

$$H^2 = \rho_1 + \rho_2 - \frac{k}{a^2} \tag{4}$$

where $k = +1, 0, -1$, corresponding to the case that the universe is spatially positive curved, flat and negative curved, respectively. In [10], s was prescribed by

$$s = -\alpha H\rho_1 + \beta H\rho_2 \tag{5}$$

Other forms of s, such as $s_A = s_0\rho a^{-1}$, $s_B = s_0 a^{-(1+3\gamma 1)}$, $s_C = s_0\Lambda$ and $s_D = s_0$ are also considered [6, 7, 11, 12]. In our previous work [21], we extended Barrow-Clifton's work to the case of a universe containing three fluids, radiation, matter and vacuum energy, with energy exchange between radiation and vacuum energy and between matter and vacuum energy.

3 Further Extension to Three Fluids: An Ansatz for Prescription of s_i and Cyclic Solution

Consider a spatially flat FRW universe containing three perfect fluids, radiation, matter and vacuum energy. The equations of state are $p_i = (\gamma_i - 1)\rho_i$, $i = 1, 2, 3$, with $\gamma_1 = 4/3$, $\gamma_2 = 1$, $\gamma_3 = 0$. Extending (2) or (3), we have

$$\dot{\rho}_1 + 4H\rho_1 = s_1 \tag{6a}$$

$$\dot{\rho}_2 + 3H\rho_2 = s_2 \tag{6b}$$

$$\dot{\rho}_3 = -(s_1 + s_2) \tag{6c}$$

or

$$\rho_1 = \frac{1}{a^4}\int s_1 a^4 \mathrm{d}t \tag{7a}$$

$$\rho_2 = \frac{1}{a^3}\int s_2 a^3 \mathrm{d}t \tag{7b}$$

$$\rho_3 = -\int (s_1 + s_2)\mathrm{d}t \tag{7c}$$

In order to find an exact cyclic solution, an ansatz, which we call "semi-inverse problem method", is used to prescribe the form of s_1 and s_2 based on the following considerations: (i) At least ρ_3 can be conveniently calculated. (ii) A periodic term should be contained in s_1 and s_2. (iii) Some duality between the forms of s_1 and s_2 should be expected. With these demands and a certain conjecture, we assume that

$$s_1 = \frac{\rho_c}{H_0^2}\left[\alpha_1 H\dot{H} - \frac{1}{4}\ddot{H} + 48(2 + \alpha_1 + \alpha_2)\frac{1}{t_0^3}\left(\frac{\sum_i^n A_i\omega_i \sin\omega_i x}{\sum_i^n A_i \cos\omega_i x}\right)^3\right] \tag{8a}$$

$$s_2 = \frac{\rho_c}{H_0^2}\left[\alpha_2 H\dot{H} - \frac{1}{3}\ddot{H} - 48(2 + \alpha_1 + \alpha_2)\frac{1}{t_0^3}\left(\frac{\sum_i^n A_i\omega_i \sin\omega_i x}{\sum_i^n A_i \cos\omega_i x}\right)^3\right] \tag{8b}$$

where $x = t/t_0$, while α_1, α_2, A_i and ω_i are all constant, and α_1 and α_2 can be determined from the cosmic dynamical equations and the observational values of the cosmological density of every component, $\Omega_{10} \equiv \rho_{10}/\rho_c$, $\Omega_{20} \equiv \rho_{20}/\rho_c$ and $\Omega_{30} \equiv \rho_{30}/\rho_c$. Firstly, from (7c), (8a) and (8b), it is easy to obtain

$$\rho_3 = \frac{\rho_c}{H_0^2}\left[-\frac{1}{2}(\alpha_1 + \alpha_2)H^2 + \frac{7}{12}\dot{H}\right] \tag{9}$$

Then, combining the cosmic dynamical equations

$$H^2 = \frac{H_0^2}{\rho_c}\rho \tag{10}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{2}\frac{H_0^2}{\rho_c}(\rho + 3p) \tag{11}$$

with the equation of state $3p = \rho_1 - 3\rho_3$, using (9) and noting $\ddot{a}/a = \dot{H} + H^2$, we can obtain

$$\rho_1 = \frac{\rho_c}{H_0^2}\left[-\frac{1}{4}\dot{H} - \frac{3}{2}(\alpha_1 + \alpha_2 + 2)H^2\right] \tag{12}$$

$$\rho_2 = \frac{\rho_c}{H_0^2}\left[-\frac{1}{3}\dot{H} + 2(\alpha_1 + \alpha_2 + 2)H^2\right] \tag{13}$$

Substituting (12) and (13) back into (6a) and (6b), and then comparing with (8a) and (8b) yield that

$$4\alpha_1 + 3\alpha_2 = -7 \tag{14}$$

$$H = -\frac{2}{t_0}\frac{\sum_i^n A_i\omega_i \sin\omega_i x}{\sum_i^n A_i \cos\omega_i x} \tag{15}$$

Integrating (15) we can find

$$a = a_0\left(\sum_i^n A_i \cos\omega_i x\right)^2 \tag{16}$$

Applying (12) or (13) to the present epoch gives the relation

$$3(\alpha_1 + \alpha_2) = -\frac{1}{4}(21 + 7\Omega_{10} + 3\Omega_{30}) \tag{17}$$

By combining (17) with (14) we can find

$$\alpha_1 = \frac{1}{4}(-7 + 7\Omega_{10} + 3\Omega_{30}) \tag{18a}$$

$$\alpha_2 = \frac{1}{3}(-7\Omega_{10} - 3\Omega_{30}) \tag{18b}$$

The cosmic observation shows that $\Omega_{10} \sim 10^{-4}$, $\Omega_{20} \approx 0.3$ and $\Omega_{30} \approx 0.7$ [22, 26, 35]. Assume that $\Omega_{30} = 0.7 - \beta\Omega_{10}$, then $\Omega_{20} = 0.3 + (\beta - 1)\Omega_{10}$ and

$$\alpha_1 = \frac{1}{4}[-4.9 + (7 - 3\beta)\Omega_{10}] \tag{19a}$$

$$\alpha_2 = \frac{1}{3}[-2.1 - (7 - 3\beta)\Omega_{10}] \tag{19b}$$

Finally, we can merge (8a) and (8b) into the united form below

$$s_i = \frac{\rho_c}{H_0^2}\left\{\frac{1}{3\gamma_i}[(2.1(3-4\gamma_i)-(7-6\gamma_i)(7-3\beta)\Omega_{10})H\dot{H}-\ddot{H}]\right.$$
$$\left.-(7-6\gamma_i)[3.6-4(7-3\beta)\Omega_{10}]\frac{1}{t_0^3}\left(\frac{\sum A_i\omega_i\sin\omega_i}{\sum A_i\cos\omega_i x}\right)^3\right\}\quad (i=1,2) \tag{20}$$

which obviously embodies the duality of s_1 and s_2.

4 A Special Solution Modeling the Real Universe

To what extent does the general solution (16) consist with our real universe? To answer this question, it is useful to find a corresponding special solution by using the cosmic observational results. Setting $n = 5$ and $\omega_i = 0.48\pi i$, then the solution (16) can be rewritten as the form

$$a = a_0\left(\sum_{i=1}^{5} A_i\cos(0.48\pi i x)\right)^2 \tag{21}$$

The conditions $a|_{x=0} = 0$ and $a|_{x=1} = a_0$ give that

$$\sum_{i=1}^{5} A_i = 0 \tag{22}$$

$$\sum_{i=1}^{5} A_i\cos(0.48\pi i) = 1 \tag{23}$$

In consideration of that $H_0^{-1} \approx 1.3\times 10^{10}$ yr and the age of the ancient galaxy is about 1.5×10^{10} yr, we can safely assume $H_0 t_0 = 3/2$. Thus, from $H|_{x=1} = H_0$ we have

$$\sum_{i=1}^{5} A_i 0.48\pi i\sin(0.48\pi i) + \frac{3}{4} = 0 \tag{24}$$

From

$$\frac{\ddot{a}}{a}|_{x=1} = -\frac{1}{2}\frac{H_0^2}{\rho_c}(\rho_0+\rho_{10}-3\rho_{30}) \approx -\frac{1}{2}(1-2.1)H_0^2,$$

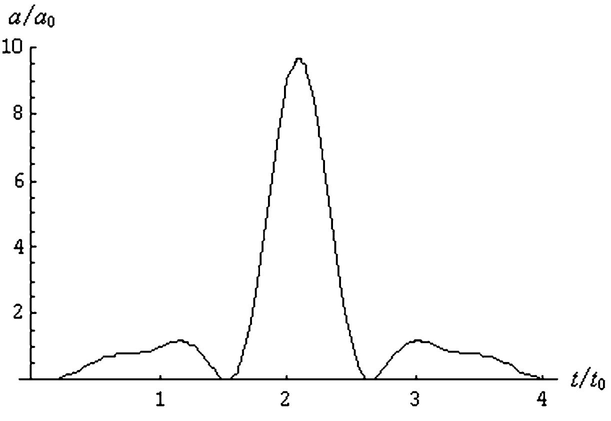
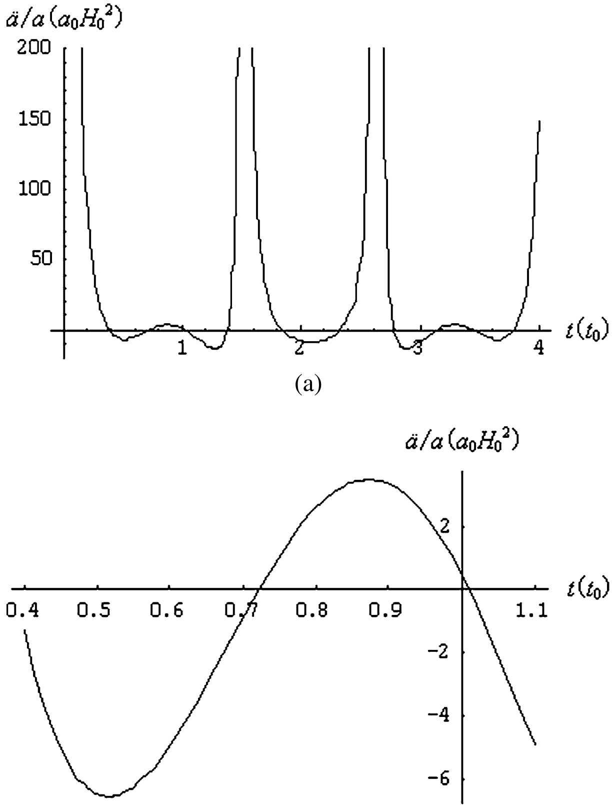


Fig. 1 The evolution of the scale factor $a(t)$ with the cosmic time t



(a)

(b)

Fig. 2 The evolution of the cosmic expansion acceleration $\ddot{a}(t)$ with the cosmological time t: (**a**) shows the cyclicity of evolution, and (**b**) shows the detail of the present epoch

it is can be obtained

$$\sum_{i=1}^{5} A_i (0.48\pi i)^2 \cos(0.48\pi i) + 0.05625 = 0 \tag{25}$$

The time at which the universe transforms from the decelerating phase to the accelerating one is denoted by t_{T}, correspondingly, $t_{\mathrm{T}}/t_0 = x_{\mathrm{T}}$. Taking the transition red-shift $Z_{\mathrm{T}} = 0.3$ [17, 27], then $\frac{a|_{x_T}}{a_0} = \frac{1}{1+Z_T} = \frac{1}{1.3}$ and $\ddot{a}|_{x_T} = 0$, which give

$$\sum_{i=1}^{5} A_i \cos(0.48\pi i x_T) = \sqrt{\frac{1}{1.3}} \tag{26}$$

$$\left(\sum_{i=1}^{5} A_i 0.48\pi i \sin(0.48\pi i x_T)\right)^2 - \sqrt{\frac{1}{1.3}} \sum_{i=1}^{5} A_i (0.48\pi i)^2 \cos(0.48\pi i x_T) = 0 \tag{27}$$

To solve the algebraic equations system (22)–(27), we find a set of numerical solution: $A_1 = 1.154514$, $A_2 = -1.292050$, $A_3 = 0.452172$, $A_4 = -0.261380$, $A_5 = -0.053256$, $x_{\mathrm{T}} = 0.707231$. With this result, the evolution of the cosmic scale factor $a(t)$ and the acceleration $\ddot{a}(t)$ as the cosmological time t can be easily obtained, which are illustrated in Figs. 1 and 2.

5 Summary

We extend the cosmological scenario with energy exchange proposed by Barrow and Clifton from the case of two fluids to the case of three fluids. We use an ansatz to prescribe exchange function with periodic term and form-dual s_1 and s_2, and find an infinitely cyclic general solution. By using the current values of some cosmological parameters, a special solution which should suit to describe our universe is found. Thus, we construct a cosmological model, in which the contribution of vacuum energy to the gravitation is embodied and an explanation of the accelerating expansion of the universe at the present epoch can be obtain without invoking any scalar field. Moreover, in our model the universe is spatially flat and undergoes an endless sequence of cosmic epoch each consisting of expansion and contraction, hence the flatness puzzle and the horizon puzzle in standard model should not exist in ours.

Acknowledgement The authors wish to thank professor Qian-Kai Yao for his useful discussions.

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